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A solution of an YDFA in tandem-pumping configuration with ASE using RK4 method

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***Abstract*—A new solution for a tandem amplifier is presented using a fiber doped with ytterbium and a core coating ratio of $30/250$ and which incorporates the solution of the rate equations with the amplified spontaneous emission characteristic in these multi-kilowatt systems. The model describes in a more precision way the behavior of a fiber amplifier with a $5.986kW$, $1018nm$ pumping and a signal seed of $75W$, $1080nm$, solving the propagation and rate equations using the fourth-order Runge-Kutta as well the relaxation methods as a convergence criterion for the integrations.**

***Index Terms*—high power fiber laser, ytterbium-doped fiber, tandem pump, propagation and rate equation model**

I. Introduction

Ytterbium doped fiber amplifiers (FA) have been developed from research laboratories for the last fifty years and it became indispensable for several components of nowadays technologies in the fields of communication, medical equipment, and materials processing etc. The use of multimode (MM) Yb^{3+} doped core double-clad cladding pumped scheme has shown great success in the development of very high power laser systems due to their high pumping power being gradually coupled to the core as it propagates through the fiber and is absorbed by the Yb^{3+}. This setup was very successful to overcome the limitations in pump power and others problems typically found in single-mode fibers amplifiers (SMFA) [1].

Although this pumping scheme has been extensively used in practice in FA, modeling rare earth doped MMF is a very demanding job in terms of computing processes. The main problem of this modeling is to effectively describe the coupling of the pump beam which has infinite modes due to the large diameter of the clad in relation to the small diameter of the core, and the effects of the coating formats that can be used affect the propagation of the beams and thus altering the pump's absorption quality and thus limiting the amplifier's performance. Undoubtedly, it is very important to understand the fiber amplification process, and also to optimize the design of laser systems that uses active FA and defines different configurations playing important roles in the laser systems performance [2-4].

In the process of high power amplification, in addition to the signal and pump fields, the Yb^{3+} ions in the excited state also emit photons in different wavelength that generates a random noise that propagates in both directions along the fiber. The so-called amplified spontaneous emission (ASE) has a great impact on the quality of the laser output, and this noise is also amplified in the propagation along the fiber, absorbing power from the pump, ruining the efficiency of the FA [5].

The modeling methods based on the solution of the propagation and rate equations have been shown to be reliable and have a high degree of precision regarding the simulation time, in the processes of testing and optimizing the performance of SMFA, however these models end up to be very demanding of computing power for MM configurations.

The application of several techniques to suppress higher order modes, such as fiber bending and use of high brightness and optical quality seed, proved to be of great importance for an effective modeling of this optical system. Since, all of these can be used so that it is possible to use the filling factor Γ_k as solution to the wavefront to replicate MM fibers, then the simulations approach the real results obtained in the laboratory.

It is also possible to model and simulate the ASE effect by solving the rate equations. The importance of simulating this effect is because this noise generates signal degradation and changes the properties of fiber amplifiers, in addition to causing damage to the device due to counter-propagation [6]. To fulfill the goal of the present work, this paper starts in section II formulating the theoretical model, and a solution for the rate equations for the amplifier with the Runge-Kutta method (RK4). In section III, a discussion is presented about the results and section IV concludes the work.

II. Theoretical Model

According to Paschotta [5], the Yb^{3+} ion spectroscopy is simpler compared to other rare earth ions. Thus, for all k wavenumbers, only the atomic sublevels ${}^2F_{\frac{7}{2}}$ (laser transition inferior level) and ${}^2F_{\frac{5}{2}}$ (metastable state) are important. In addition, the population thermalization within each sub-level is so fast that it can be considered instantaneous; therefore, it is only necessary to specify the populations N_1 and N_2 at the lowest and highest sublevels, respectively.

$$N_t\left(z,t\right) = N_1\left(z,t\right) + N_2\left(z,t\right). \tag{1}$$

Assuming that the amplifier is working in the steady state dN_0/dt is none. the sum of the variation of populations N_1 and N_2 is always constant, so

$$\frac{d}{dt}\left(N_1+N_2\right)=0\rightarrow\frac{dN_1}{dt}=-\frac{dN_2}{dt}, \tag{2}$$

and the rate of changing of the highest sublevel population dN_2/dt will be dependent on the phenomena of stimulated emission (e), spontaneous emission (ν) and absorption (a). The rate equations depend of the total dopant concentration N_t, the wave frequency ν_k, the radiative lifetime τ_2 and the Planck's constant h. They are described by Giles [7] as follows:

$$\frac{dN_2}{dt}=-\left[R_p^e+W_{s,\nu}^e+\frac{1}{\tau_{21}}\right]N_2+\left[W_{s,\nu}^a+R_p^a\right]N_1. \tag{3}$$

$$\frac{N_2}{N_t}=\frac{R_p^a+W_{s,\nu}^a}{R_p^a+R_p^e+W_{s,\nu}^a+W_{s,\nu}^e+\frac{1}{\tau_{21}}}. \tag{4}$$

where R_p^a is the pump absorption rate, R_p^e is the pump emission rate, $W_{s,\nu}^a$ is the signal and ASE photons absorption rates, and $W_{s,\nu}^e$ is the emission rate for signal and ASE, as follows:

$$R_p^a=\frac{\Gamma_p\cdot\sigma_p^a}{A_{eff}\cdot h\nu_p}\cdot\left[P_p^++P_p^-\right], \tag{5}$$

$$R_p^e=\frac{\Gamma_p\cdot\sigma_p^e}{A_{eff}\cdot h\nu_p}\cdot\left[P_p^++P_p^-\right], \tag{6}$$

$$\begin{aligned}W_{s,\nu}^a=&\frac{\Gamma_s\cdot\sigma_s^a}{A_{eff}\cdot h\nu_s}\cdot\left[P_s^++P_s^-\right]\\&+\sum\frac{\Gamma_\nu\cdot\sigma_\nu^a}{A_{eff}\cdot h\nu}\cdot\left[P_\nu^++P_\nu^-\right]\cdot\Delta\nu\end{aligned}, and \tag{7}$$

$$\begin{aligned}W_{s,\nu}^e=&\frac{\Gamma_s\cdot\sigma_s^e}{A_{eff}\cdot h\nu_s}\cdot\left[P_s^++P_s^-\right]\\&+\sum\frac{\Gamma_\nu\cdot\sigma_\nu^e}{A_{eff}\cdot h\nu}\cdot\left[P_\nu^++P_\nu^-\right]\cdot\Delta\nu\end{aligned}. \tag{8}$$

A. Basic Rate Equations

Expanding Eq. (3) and taking into account the three types of light power, the signal $k=\{s\}$, the pumping $k=\{p\}$ beams, and the ASE $k=\{\nu\}$ beams. The $+$ standing for the co-propagation and the $-$ for contra-propagation. The signal laser P_s^+ and the pumping laser $P_p^\pm$ have different spectral properties each, and the pumping laser can travel in both directions, depending on the experiment configuration (single-pumping or dual-pumping). Assuming that the laser is working in continuous wave mode (CW), and after the end of the initial transient, the rate of change dN_2/dt is none, and Eq. (4), as seen above, is obtained. The propagation of the signal and pumping lasers along the fiber are described by Eqs. (9)-(11):

$$\frac{dP_p^\pm}{dz}=\pm\Gamma_p\cdot\left[\left(\sigma_p^e+\sigma_p^a\right)\cdot N_2-\sigma_p^a\cdot N_t\right]\cdot P_p^\pm\pm\alpha_p\cdot P_p^\pm, \tag{9}$$

$$\begin{aligned}\frac{dP_s^\pm}{dz}=&\pm\Gamma_s\cdot\left[\left(\sigma_s^e+\sigma_s^a\right)\cdot N_2-\sigma_s^a\cdot N_t\right]\cdot P_s^\pm\\&\mp\alpha_s\cdot P_s^\pm\end{aligned}, and \tag{10}$$

$$\begin{aligned}\frac{dP_\nu^\pm}{dz}=&\pm\Gamma_\nu\cdot\left[\left(\sigma_\nu^e+\sigma_\nu^a\right)\cdot N_2-\sigma_\nu^a\cdot N_t\right]\cdot P_\nu^\pm\\&\mp\alpha_\nu\cdot P_\nu^\pm\pm\Gamma_\nu\sigma_\nu^e h\nu\Delta\nu N_2\end{aligned}. \tag{11}$$

These differential equations are a function of the filling factor Γ_k and the cross sections $\sigma_k^{a,e}$, representing the gain, and the term α_k representing the power losses.

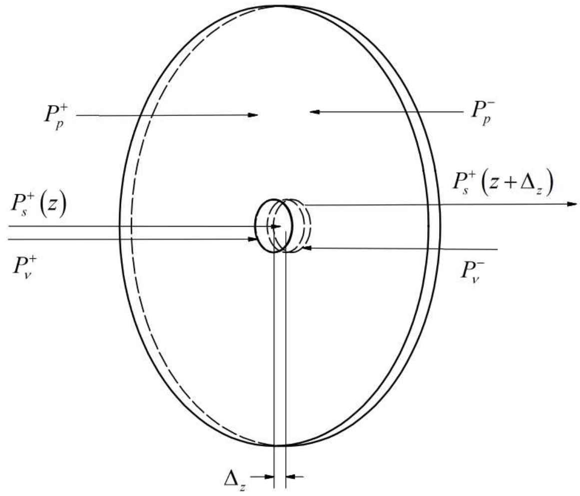

Fig. 1. Schematic Beam Propagation Method (BPM) for a cladding-pumped Yb^{3+} doped fiber amplifier.

The diagram in Fig. 1 shows the beam propagation method for a disk of diameter Δ_z for fiber amplifiers pumped by the cladding. At the input of fiber z, the pump powers $P_p^\pm$, the signal $P_s^+(z)$ and the ASE $P_\nu^\pm$ are the system input. With the solution of the rate equations it is possible to determine the population $N_2(z)$ and then calculate back the powers with the new gain parameters $\pm\Gamma_k\cdot[(\sigma_k^e+\sigma_k^a)\cdot N_2-\sigma_k^a\cdot N_t]$ and with the absorption factors α_ν at that position n. In this way it is then possible to integrate by RK4 the new optical powers that will then be used in the BPM to propagate the beam with a step Δ_z. This process is repeated until the end of the fiber length when $z=L$.

B. Fourth-Order Runge-Kutta Model

In his book, Desurvire showed that $\Delta\nu_i=0.5nm$ is a small enough range to obtain good simulation results compared to the computation time spent [2]. Since care must be taken with the number of equations to be solved as they are proportional to this discretization width of the spectrum. For the spectrum that is divided into n intervals, the number of equations that must be solved will be $(2n+4)$. The coefficient 2 is due to ASE noise propagation back and forth in each slot.

The first order ordinary differential equations (ODE) Eqs. (9)-(11) have a single independent variable z. Thus, the Runge-Kutta method can be used to solve this set of equations. RK4 solves initial value problems by integrating its solutions through multiple summations. Given the following ODE for the $P_k^\pm$ beam propagation:

$$\frac{dP_k^\pm}{dz}=f\left(z,P_k^\pm\right), \tag{12}$$

with known initial conditions $z(0)$ and $P_k^{\pm}(0)$. When applying the RK4 method its important initially to choose a suitable step interval Δ_z which is the next point at which the equation will be integrated $z + \Delta_z$ and find the corresponding value of the function at this point using the following relations:

$$z_{n+1} = z_n + \Delta_z. \tag{13}$$

$$P^{\pm}_{k\ n+1} = P^{\pm}_{k\ n} + \Delta_k. \tag{14}$$

where,

$$k_1 = \Delta_z \cdot f\left(z_n, P^{\pm}_{k\ n}\right). \tag{15}$$

$$k_2 = \Delta_z \cdot f\left(z_n + \frac{\Delta_z}{2}, P^{\pm}_{k\ n} + \frac{k_1}{2}\right). \tag{16}$$

$$k_3 = \Delta_z \cdot f\left(z_n + \frac{\Delta_z}{2}, P^{\pm}_{k\ n} + \frac{k_2}{2}\right). \tag{17}$$

$$k_4 = \Delta_z \cdot f\left(z_n + \Delta_z, P^{\pm}_{k\ n} + k_3\right). \tag{18}$$

$$\Delta_k = \frac{1}{6}\left(k_1 + 2\left(k_2 + k_3\right) + k_4\right). \tag{19}$$

Fig. 2 shows the flow chart used to simulate a FA with all propagating waves already represented. The shooting method was used since at first it is not possible to know the general shape of the frequency spectrum of the output signal, in this way some arbitrary distribution can be assumed according to Meyer, Sompo and Solms [8]. First, it is necessary to set

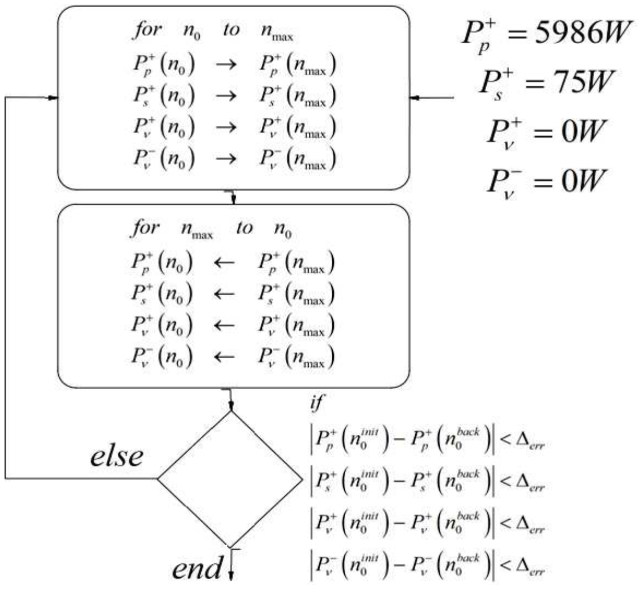

Fig. 2. Schematic simulation for a cladding-pumped Yb^{3+} doped fiber amplifiers with ASE.

the size of the longitudinal section so that $z(n_{max}) = L$. Then using the initial values for the direct pump power $P_p^+ = 5986W$, signal power $P_s^+ = 75W$, and setting the noise $P_\nu^+ = 0W$ and an estimate value was set for the noise back as $P_\nu^- = 0W$ at $z = 0$, the equations are integrated in the forward direction until $z(n_{max}) = L$. In the next step, the propagation equations are integrated in the reverse direction also using the RK4 scheme in each section up to $z(n_0) = 0$. This algorithm is a loop where the new value of P_ν^- is then compared with the initial from boundary condition until the error is within the defined range.

C. Relaxation method

Initially the values for the noise was set to zero $P_\nu^{\pm} = 0W$ at the first run of the loop, this is the shooting method, although this criteria may take a long time to converge the system the ODE‘s it is possible to have a some guess for this initial value problem. Then, using the Relaxation method which is:

$$P^+_{\nu\ new} := \lambda \times P^+_{\nu\ new} + (1-\lambda) \times P^+_{\nu\ old}; \tag{20}$$

$$P^-_{\nu\ new} := \lambda \times P^-_{\nu\ new} + (1-\lambda) \times P^-_{\nu\ old}; \tag{21}$$

if

$$\left|\frac{P_p^+\left(n_0^{init}\right) - P_p^+\left(n_0^{back}\right)}{P_p^+\left(n_0^{init}\right)}\right| < \Delta_{err} \tag{22}$$

&&

$$\left|\frac{P_s^+\left(n_0^{init}\right) - P_s^+\left(n_0^{back}\right)}{P_s^+\left(n_0^{init}\right)}\right| < \Delta_{err}; \tag{23}$$

where λ is the relaxation factor. So, instead of guessing a new value using a method like taking the average of the initial and the final value or using the secant method or any other. This method takes the final value obtained after the first forward and backward integration, and apply a weight as it it shown in Eqs. (20) and (21). The use of the relaxation method accelerates the rate of convergence and proved to be faster than the guessing method, it is a good choice under general conditions [8].

III. Results and Discussion

The parameters in Table I were used to run the computational code developed in MATLAB to solve the rate equations described previously. These data were obtained from Yan [9] and [10].

TABLE I
Fiber and Thermal Parameters Used for Simulating [9] and [10].

Symbol	Parameter	Value
YDF	Yb^{+3} doped fiber	$30/250\mu m$
A_{eff}	Effective core area	$1.6997 \times 10^{-9} m^2$
L	Fiber length	$38m$
NA	Numerical Aperture	$0.06/0.46$
$P_p^+(z)$	Pumping power (PP)	$5986W$
$P_s^+(z)$	Seed power (SP)	$75W$
λ_s	(SP) wavelength	$1.080\mu m$
λ_p	(PP) wavelength	$1.018\mu m$
Γ_s	Filling factor (SP)	0.82
Γ_p	Filling factor (PP)	0.0144
N_t	Yb^{+3} concentration	$5.885 \times 10^{25} m^{-3}$
τ_2	Radiative lifetime	$1ms$
σ_p^a	(PP) absorption cross section	$74.6 \times 10^{-27} m^{-2}$
σ_p^e	(PP) emission cross section	$580.0 \times 10^{-27} m^{-2}$
σ_s^a	(SP) absorption cross section	$2.30 \times 10^{-27} m^{-2}$
σ_s^e	(SP) emission cross section	$282.0 \times 10^{-27} m^{-2}$
α_p	Scattering loss coefficient (PP)	$2.0 \times 10^{-3} m^{-1}$
α_s	Scattering loss coefficient (SP)	$2.0 \times 10^{-3} m^{-1}$

Fig. 3 shows the evolution of forward and backward signal, pump and ASE powers, which were plotted according to the solution of the equations shown above.

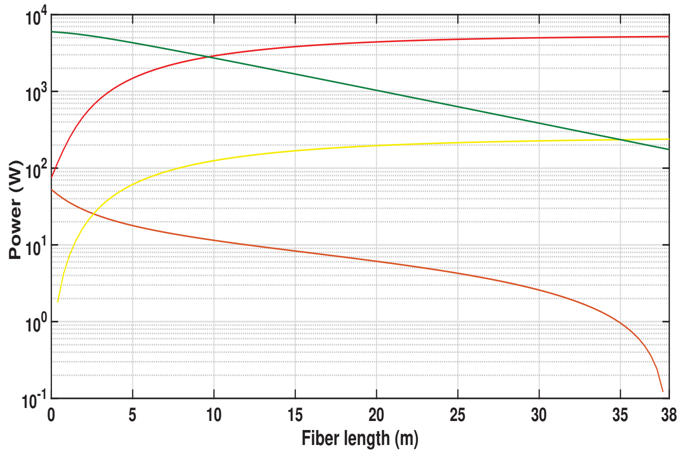

Fig. 3. $P_s^+(z)$ (RED), $P_\nu^+(z)$ (YELLOW), $P_\nu^-(z)$ (ORANGE) and $P_p^+(z)$ (GREEN).

Comparing the results obtained with those published by Yan [9] it is possible to verify that the evolution of the signal power along the axial length when compared with the experimental data, it is possible to evaluate the average error which is around 5%. This difference in the results, although small, behaved proportionally to the model curve shown in Fig. 4

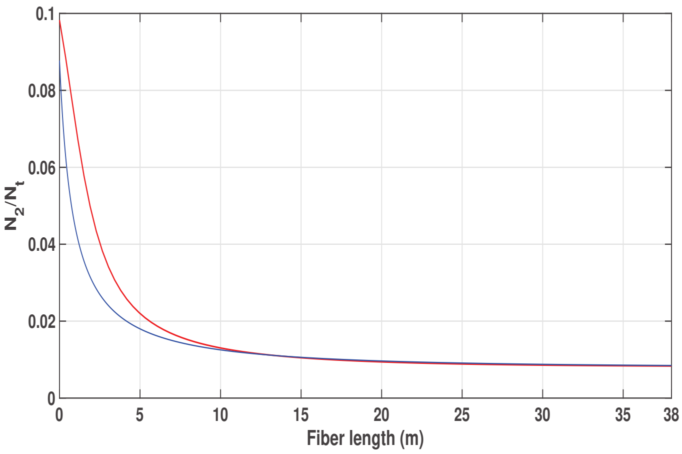

Fig. 4. $N_2(z)$ to $/N_t ratio$ model (RED) and experimental (BLUE).

It was possible to obtain the axial distribution of the population N_2, Fig. 4 shows this excited axial distribution Yb^{3+} obtained from our model (red), which agrees with the data presented in the works of Yan [9] and Wang [11] (blue). These two curves have the same order of magnitude, and as can be seen, the error at the beginning of the fiber has a very high rate, but it decreases very quickly. This difference can be explained by the fact that part of the pumping power is being absorbed by the noise, and the power for the direct noise was $P_\nu^+ = 238.29W$, so the simulated output power was $5417.77W$ which is in agreement with our previous work at a ratio of approximately 0.5% [12].

Another important point to be observed is that the simulated return noise was $P_\nu^- = 52.7W$, which means that it is essential to pay attention to the backward propagation and its deleterious effect to the signal input, the use of an optical isolator can be critical to avoid any damage to the seed laser.

IV. Conclusion

In this study, a solution for the Beam Propagation Method using fourth-order Runge-Kutta method with the Relaxation Method was described. It was effective to simulate a multi-kilowatt ytterbium-doped double-cladding tandem-pumped fiber amplifier, solving the signal power, pumping power, and ASE propagation and the variation of the upper-level population along the full axial length of the active fiber, integrating those equations back and forth until the change of the entry parameters got below the given error. This method has the benefit of a high speed of convergence and robustness. An advantage of this approach is that it facilitates the optimization in the designing of fiber amplifiers because it makes easy to change parameters like length, dopant concentration, output power, and efficiency. It was observed that the model results are in a good agreement with the work of Yan [9], with a maximum error under -5.4%. It indicates that the noise amplified in both directions can cause degradation to the output signal and the increase of back propagation of the noise must be suppressed in order to minimize the risk of damaging the laser system.

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