EVALUATION OF EXTRAPOLATED RANGE OF ELECTRONS FOR LIGHT ELEMENTS ($Z \leq 22$) USING A NEW GENERALIZED EMPIRICAL EQUATION FOR THE ELEMENTAL AND COMPOUND MATERIALS

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ABSTRACT

The extrapolated range R_{ex} of electrons is useful for various purposes in research and the utilization of electrons, for example in polymer modification, electron energy determination and estimation of effects associated with deep penetration of electrons. A number of works have used empirical equations to express the extrapolated range. A generalized empirical equation, very simple and accurate, for the extrapolated or practical range R_{ex} of monoenergetic electrons in the energy region 0,3keV - 50MeV, and for the elemental absorbers of atomic numbers 2 - 92, was proposed by the authors. It was also proposed a simplified equation for light elements ($Z \leq 22$). In this work, the extrapolated range for light elements using that equation is evaluated and the results are compared with literature experimental data.

The extrapolated or practical range R_{ex} of monoenergetic electrons in the energy region 0,3keV - 50MeV for the elemental absorbers of atomic numbers 2 - 92 was found to be well expressed by the following equation (paper to be published):

$$\boxed{R_{ex} = 1.41\frac{Z^{0.68}}{Z+2}E\left[1-\frac{0.985}{1+Z^{1.9}\cdot 10^{-5}+3.1E}\right]-Z^{0.45}E^{2.12}\cdot 10^{-4}} \qquad \text{(Equation 1)}$$

where: R_{ex} is the extrapolated or practical range in g/cm^2; E is the incident electron energy in MeV and Z is the atomic number.

The same equation applies to organic and inorganic molecules since we express Z by a parameter Z_M defined by:

$$Z_M = \frac{\sum_i N_i Z_i + 4N_H}{\sum_i N_i} \qquad \text{(Equation 2)}$$

where: N_i is the number of atoms i in the molecule; Z_i is the atomic number of atom i and N_H is the number of hydrogen atoms in the molecule. For example:

1. Polyethylene

$$C_2H_4 \Rightarrow Z_M = \frac{12+16}{2} = 14$$

2. Water $H_2O \Rightarrow Z_M = \dfrac{8+8}{1} = 16$

For compound materials we need to replace Z by:

$$Z_C = \frac{\sum_i \frac{fi}{M_i} ZM_i}{\sum_i \frac{f_i}{M_i}} \qquad \text{(Equation 3)}$$

where: f_i is the fraction by weight of the molecule i;

$\dfrac{fi}{Mi}$ = molar proportion of molecule i

$Z_{Mi} = Z_m$ of molecule i.

For example: Cellulose triacetate (CTA)

($C_{12}H_{16}O_8$) 85% $\Rightarrow Z_{M1} = 10$; $f_1 = 0.85$; $M_1 = 288$

($C_{18}H_{15}PO_4$) 15% $\Rightarrow Z_{M2} = 9.35$; $f_2 = 0.15$; $M_2 = 326$

Calculating by the Eq. 3, we have: $Z_C(\text{CTA}) = 9.91$

If we have only the elemental composition in the fraction by weight, then Z_M is:

$$Z_M = \frac{\sum_i \frac{fi}{Ai} Z_i + 4 f_H.}{\sum_i \frac{fi}{Ai}} \qquad (2.a)$$

Where: fi = fraction by weight of atom i; f_H = fraction by weight of hydrogen atom; A_i = atomic mass of atom i.

For example: Bone (tissue), composition: 0.064 H; 0.278 C; 0.027 N; 0.410 O; 0.002 Mg; 0.070 P; 0.002 S; 0.147 Ca.
Calculating by Eq. (2.a), we have: Z_M (bone) = 12.74

The term $Z^{1.9} . 10^{-5}$ in Eq. 1 acts only for high Z and electron energies below 100keV, and it was inserted to increase the extrapolated range for heavy elements in this region of energy, because the stopping power decreases due to elastic electron - atom collisions. Then, for light materials (Z or Z_M < 20) R_{ex} can be expressed by:

$$\boxed{R_{ex} = 1.41 \frac{Z^{0.68}}{Z+2} E \left[1 - \frac{0.985}{1+3.1E}\right] - Z^{0.45} E^{2.12} .10^{-4}} \qquad \text{(Equation 4)}$$

On the other hand, the second term in Eq. 1 is a correction related to radiation energy losses (bremsstrahlung production), and can be neglected for low Z or Z_M materials at energies below 10MeV (for Al, Z=13 and for 10MeV, this correction being less than 1%). Then, for E ≤ 10MeV and light materials, the following equation can be used:

$$\boxed{R_{ex} = 1.41 \frac{Z^{0.68}}{Z+2} E \left[1 - \frac{0.985}{1+3.1E}\right]}$$

(Equation 5)

This representation has the same form of Weber(1) semiempirical equation (Eq. 6) for Al in the range of 3keV - 3MeV, and was corrected by Kobetich and Katz(2) for application in the range of 3keV - 20MeV, only for Al.

$$R_{ex} = AE\left[1 - B(1+CE)^{-1}\right]$$

(Equation 6)

The constants B and C are almost the same and the constant A is now a Z or Z_M function in Equation 5.

Since industrial electron beam applications are limited to light materials and beam energies lower than 10 MeV, the equation 5 is very useful, simple and easy.

In fig. 1 we compare the eq. 5 with the experimental results taken from literature (2), (4), (5), (7), (8), (9), (10) and (11). For water, data was taken from Tabata's generalized semiempirical equation (3) and corrected to fit recent Monte Carlo results(6),for A-150 tissue equivalent plastic, the data was scaled from continuous-slowing-down range r_0 (12) by:
$R_{A\text{-}150} = ((r_0/\rho)_{A\text{-}150} / (r_0/\rho)_{water}). R_{water}$

REFERENCES

(1) WEBER, K. H. N. Eine einfache reichweite - energie - beziehung für eletronen im energiebereic von 3eV bis 3MeV. **Nuclear Instruments and Methods**., **25**: 261-264, 1964.

(2) KOBETICH, E. J.; KATZ, R. Energy deposition by electron beams and γ rays. **Physical Review**, **170**(2): 391-396, 1968.

(3) TABATA, T.; ITO, R. OKABE, S. Generalized semiempirical equations for the extrapolated range of electrons. **Nuclear Instruments and Methods**, **103**: 85-91, 1972.

(4) AGU, B. N. C.; BURDETT, T. A.; MATSUKAWA, . E. The transmission of electrons through metalic foils. **Proc. Phys. Soc. (London)**, **72**: 727-732, 1958

(5) SELINGER, H. H. Transmission of positrons and electrons. **Physical Review**. **100**(4): 1029-1037,1955

(6) TABATA, T.; ANDREO, P.; ITO, R. Analytic fits to Monte Carlo calculated depth-dose curves of 1- to 50-MeV electrons in water. **Nucl. Instr. And Meth. In Physics Research**, B58: 205-210, 1991.

(7) KOBETICH, E.J.; KATZ, R. Electron energy dissipation. **Nuclear Instruments and Methods**. **71**, 226-230, 1969.

(8) HARDER, D.; POSCHET, G. Tansmission und reichweite schneller elektronen im energiebereich 4 bis 30 MeV. **Physics Letters**, **24B**(10), 519-521, 1967.

(9) TABATA, T.; ITO, R.; OKABE, S; FUJITA, Y.. Extrapolated and projected ranges of 4- to 24-MeV electrons in elemental materials. **Journal of Applied Physics**, **42**(9), 3361-3366.

(10) LOCKWOOD, G.J.; MILLER, G.H.; HALBLEID, J. A. Electron beam deposition in multilayer geometries. **IEE Trans. Nuclear Science**, **NS-15**(5), 1862-1866, 1976.

(11) TABATA,T.; SHINODA,K.;ANDREO,P.;CHUAN-SAN,W.;ITO,R. Analysis of Monte-Carlo depth-dose data for electron beams and four layer extension of the Edmult Code. **RADTECH Asia**, 574-579, 1993.

(12)ICRU - Radiation dosimetry electron beams with energies between 1 and 50 MeV, ICRU - Rep. 35, 1984.

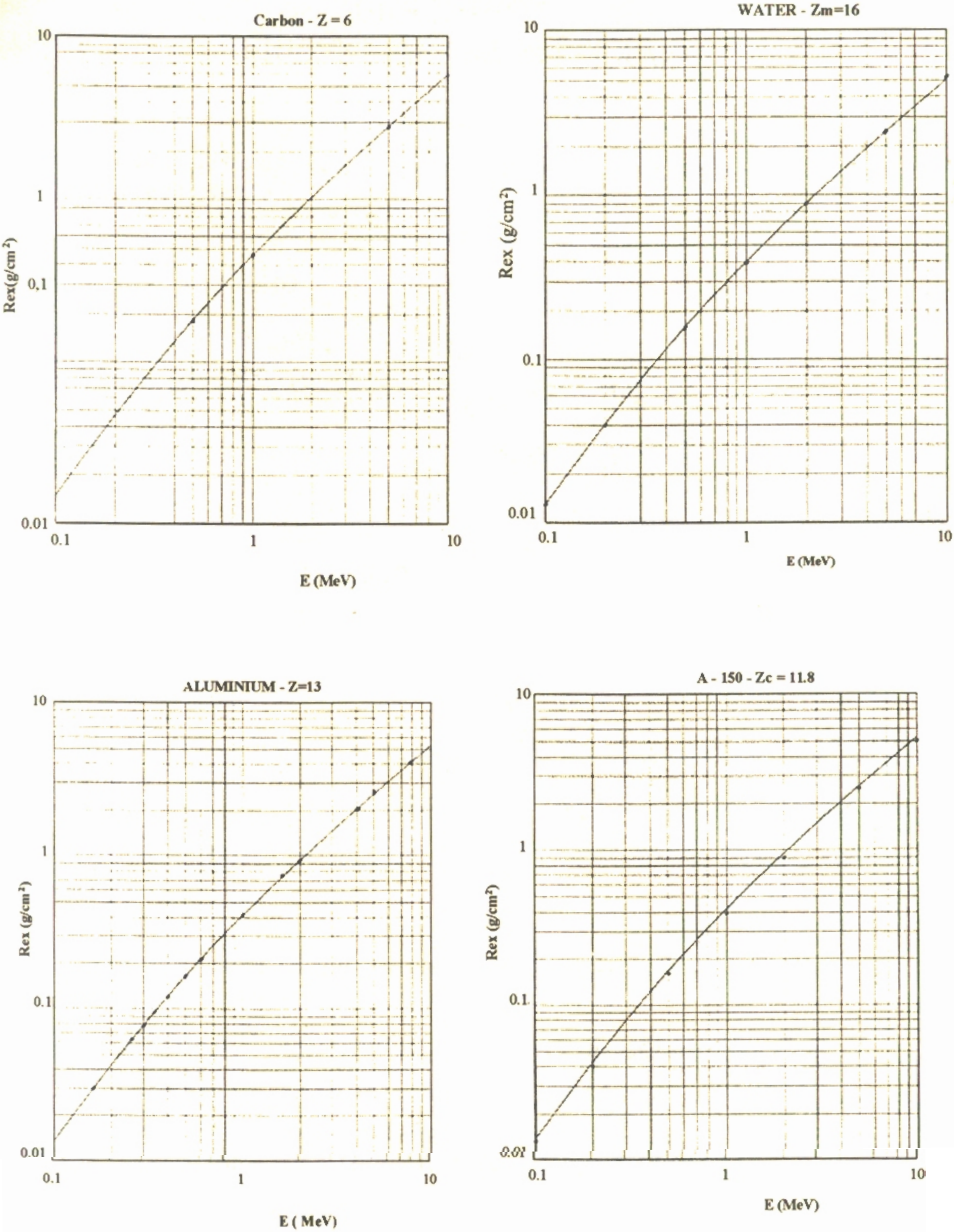

Figure1. Extrapolated range for electrons in light materials, between 100 keV and 10 MeV. Line - equation 5, points - experimental data.